Example 1

1. *Show that for large size parameters and in the limit of weak absorption and refraction the single scattering albedo is linearly related to the effective radius,* r_{eff}*, by*

$$1 - \omega \approx \frac{8\pi |m_i|}{3\lambda} r_{eff}$$

where λ *is the wavelength and* m_i *is the imaginary part of the index of refraction.*

Geometric optics limit is $x \gg 1$, weak refraction limit is $m - 1 \ll 1$, and weak absorption limit is $m_i x \ll 1$.
Using the concept of penetration depth in a bulk material we find that the transmission of a ray through a particle is

$$\mathcal{T} = \exp(-\beta_a d)$$

where $\beta_a = 4\pi m_i/\lambda$ is the bulk material absorption coefficient and d is the distance the ray travels through the particle. In the limit of weak absorption the transmission is near 1, and the absorptance fraction of the ray is $a = \beta_a d$.
To find the total absorption cross section from all rays entering a sphere, we need to integrate over the cross sectional area of the sphere. In the limit of weak refraction the rays simply travel straight through the sphere (no bending). Thus the integral of the distance through the sphere is simply the volume:

$$C_{abs} = \beta_a \int_{area} d\, dA = \beta_a V = \beta_a \frac{4\pi r^3}{3}$$

The single scattering coalbedo for a distribution of droplets is

$$1 - \omega = \frac{\int C_{abs} n(r) dr}{\int C_{ext} n(r) dr} \; .$$

In the geometric optics limit the extinction is $C_{ext} = Q_{ext} \pi r^2 = 2\pi r^2$. The single scattering coalbedo is then

$$1 - \omega = \frac{2\beta_a}{3} \frac{\int r^3 n(r) dr}{\int r^2 n(r) dr}$$

$$1 - \omega = \frac{2\beta_a}{3} r_{eff} \;\; = \frac{8\pi m_i}{3\lambda} r_{eff}$$